International Journal of Mathematics, Statistics and Operations Research
Vol. 5; Number 2; 2025, pp. 223-231
https://doi.org/10.47509/IJMSOR.2025.v05i02.03

ARF INDIA
Academic Open Access Publishing
www.arfjournals.com

Application of the Taki Integral Transform in Medical Sciences

A. K. Thakur[a], DEEPAK KUMAR[a]

[a] *Department of Mathematics, Guru Ghasidas Vishwavidyalaya, Bilaspur, India.*

ABSTRACT

Many scholars nowadays have proposed numerous integral transformations. Most often, integral transformations are used to solve systems, differential equations, and integral-differential equations. Rachid and Patil introduced the Taki integral transform in (2025). A set of differential equations with boundary conditions is present in our health sciences medication absorption models. The Taki integral transform is used in this study to solve the drug absorption model.

KEYWORDS

Integral equations, Integral transforms, Differential equations, Taki integral transform, Drug absorption, Health science.

1. Introduction

We always keep searching for new ways to make human life easier. Many problems also come up, we try to find ways to solve them, many ways are also found, but it is not necessary that that particular way is the best. We try to find the easiest ways. Like problems like engineering, physical, social, psychological, health science, financial, management etc. we take the help of technology to solve them. Now a days many problems related to human health are faced, diseases come, for their treatment we use some medicines, which we eat or apply in blood, and how it spreads in the body, I have done a lot of mathematical research on this problem In present, integral transforms are one of the very valuable and humble mathematical technique for finding the solutions of advance problems followed in many areas like, economics Engineering, commerce, technology and science. To gives exact solution of problem without long calculations is the significant feature of integral transforms.

In 2017, Khanday et al. introduced the two models shown below [1]. They use the Laplace transform and the Eigen value approach to solve these models. The relationship between medication intake and concentration at the target location across several compartments in the biological approach is examined while models are being raised. In 2022, D.P. Patil and colleagues used the General Integral Transform to solve the same model [2] Bugami, et al. (2024) using El-zaki tansform to solve blood glucose concentration model[4], A.K.Thakur and M. Kulmitra use Laplace Transform for solving the Partial Differential Equations of triple variables [5], Jaiswal, Yashawant, et al.

CONTACT Author[c]. Email: deepak.tandon1995@ggu.ac.in

Article History

Received : 11 August 2025; Revised : 09 September 2025; Accepted : 19 September 2025; Published : 10 November 2025

To cite this paper

A. K. Thakur, & Deepak Kumar (2025). Application of the Taki Integral Transform in Medical Sciences. *International Journal of Mathematics, Statistics and Operations Research.* 5(2), 223-231.

solve singular pseudo-hyperbolic equation using decompositions method using Triple generalized Sumudu Laplace transform [6], A.K. Thakur et al. to solve laplace transform application through fractional differential equation [7], Saadeh et al. use to solve cancer models through integral transform [9], A. K. Thakur and S. Panda study some properties of triple Laplace transform [8]
This paper contents four sections, first section is introduction, here we give some introduction abut over paper, second section is Preliminaries, here we say about some basic concepts, third section is application part we use Taki integral transfor [3] to solve drug absorption mode and we draw graph with using matlab and fourth section is over conclusion.

2. Preliminaries

Presenting a comprehensive integral transform that includes most integral transforms from the Laplace transform family is the aim of this section.

Definition 2.1. We consider functions of exponential order in the set $\mathcal{B}$, defined by

$$\mathcal{B} = \left\{ f(t) with |f(t)| < \mathcal{N} e^{\frac{|t|}{S_j}}, if\ t \in (-1)^j \times [0, \infty) with j = 1, 2\ and\ \mathcal{N}, s_1, s_2 > 0 \right\}, \tag{2.1}$$

For a given function in the set $\mathcal{B}$, the constant $\mathcal{N}$ must be a finite number, while g_1 and g_2 may be finite or infinite. The TIT denoted by the operator $TK(.)$, defined by the integral equation:

$$TK\{f(t)\} = \int_0^\infty R(t, v) f(\mathcal{Z}t) dt, R(t, v) = v^{\hat{\alpha}} e^{-v^{\hat{\beta}} t}. \tag{2.2}$$

where $K(t, v)$ is kernel of transformation, with $t > 0, h_1 < v < h_2, \hat{\alpha} \in \mathbb{Z}, \hat{\beta} \in \mathbb{Z}^*$ and $\mathcal{Z}$ is a real number.The inverse of the TIT is denoted by $TK^{-1}(.)$ [3].

Definition 2.2. If $TK[f(t)] = F(v)$ then the inverse of the TIT defined as

$$TK^{-1}[F(v)] = f(t).$$

.[3]

2.1. *TIT of Some functions and it's properties*

2.2. *The linearity property of TIT*

$TK[f(t)] = F[v]$ and $TK[g(t)] = G[v]$ are two TIT, then

$$TK\{\gamma_3 f(t) + \gamma_4 g(t)\} = \gamma_3[F(v)] + \gamma_4 G[v]$$

More over, the inverse TIT is linear.
If
$TK^{-1}[F(v)] = f(t)$
$TK^{-1}[G(v)] = g(t)$

Then

$$\begin{aligned} TK^{-1}[\gamma_3 F(v) + \gamma_2 G(v)] &= \gamma_3 TK^{-1}[F(v)] + \gamma_4 TK^{-1}[G(v)] \\ &= \gamma_3 f(t) + \gamma_4 g(t) \end{aligned}$$

2.3. *TIT of some important function:*

The TIT [3] is necessary to satisfy the following condition $t > 0$, we can writer it in the following form

Table 1.: TIT of some function's

s.no.	Functions f(t)	TK(f(t)) =F(v)
1	1	$\frac{\frac{v^{\hat{\alpha}}}{\mathcal{Z}}}{\frac{v^{\hat{\beta}}}{\mathcal{Z}}}$
2	t	$\mathcal{Z}\frac{v^{\hat{\alpha}}}{v^{2\hat{\beta}}}$
3	t^2	$2\mathcal{Z}^2\frac{v^{\hat{\alpha}}}{s^{3\hat{\beta}}}$
4	t^n	$n!\mathcal{Z}^n\frac{v^{\hat{\alpha}}}{v^{(n+1)\hat{\beta}}}$, where $n \in \mathbb{N}^*$
5	e^{at}	$\frac{P^{\hat{\alpha}}}{s^{\hat{\beta}}-a}$
6	sinat	$\frac{a\frac{v^{\hat{\alpha}}}{\mathcal{Z}}}{\{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\}^2+a^2}$
7	cosat	$\frac{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\frac{s^{\hat{\alpha}}}{\mathcal{Z}}}{\{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\}^2+a^2}$
8	sinhat	$\frac{a\frac{v^{\hat{\alpha}}}{\mathcal{Z}}}{\{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\}^2-a^2}$
9	coshat	$\frac{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\frac{v^{\hat{\alpha}}}{\mathcal{Z}}}{\{\frac{v^{\hat{\beta}}}{\mathcal{Z}}\}^2-a^2}$

2.4. *TIT of derivative*

Let f(t) be a function derivable in R, the TIT of its derivative is written as follows:

$$1. f'(t) \implies TK[f'(t)] = \frac{v^{\hat{\beta}}}{\mathcal{Z}} TK[f(t)] - \frac{v^{\hat{\alpha}}}{\mathcal{Z}} f(0)$$

$$2. f''(t) \implies TK[f''(t)] = (\frac{v^{\hat{\beta}}}{\mathcal{Z}})^2 TK[f(t)] - \frac{v^{\hat{\beta}}}{\mathcal{Z}}\frac{v^{\hat{\alpha}}}{\mathcal{Z}} f(0) - \frac{v^{\hat{\alpha}}}{\mathcal{Z}} f'(0)$$

$$3. f^{(n)}(t) \implies TK[f^{(n)}(t)] = (\frac{v^{\hat{\beta}}}{\mathcal{Z}})^n TK[f(t)] - \frac{v^{\hat{\alpha}}}{\mathcal{Z}} \sum_{k=0}^{n-1} (\frac{v^{\hat{\beta}}}{\mathcal{Z}})^{n-1-k} f^{(k)}(0)$$

[3]

3. Application

medication absorption and circulation via the blood and gastrointestinal system using a two-compartment model. The normal form of the two compartment model defining the rate of change in oral drug management is given as [1],

$$\begin{cases} \frac{dl_1(t)}{dt} = -m_1 l_1(t) \\ \frac{dl_2(t)}{dt} = m_1 l_1(t) - m_2 l_2(t), \end{cases} \quad l_1(0) = l_0, l_2(t) = 0 \tag{3.1}$$

Here $l_1(t)$: Concentration of drug in stomach or gastrointestinal tract,
$l_2(t)$: Concentration of drug in bloodstream sections,
l_0: Initial concentration of blood,
m_1: Rate constants from one compartment to another,
$m_2(>0)$: Clearance constant.

apply Taki integral transform in euation (3.1)

$$\begin{cases} TK[\frac{dl_1(t)}{dt}] = TK[-m_1 l_1(t)] \\ TK[\frac{dl_2(t)}{dt}] = TK[m_1 l_1(t) - m_2 l_2(t)], \end{cases} \tag{3.2}$$

$$\begin{cases} \frac{v^{\hat{\beta}}}{z} TK[l_1(t)] - \frac{v^{\hat{\alpha}}}{z} l_1(0) = -m_1 TK[l_1(t)] \\ \frac{v^{\hat{\beta}}}{z} TK[l_2(t)] - \frac{v^{\hat{\alpha}}}{z} l_2(0) = m_1 TK[l_1(t)] - m_2 TK[l_2(t)], \end{cases} \tag{3.3}$$

using initial conditions and simplifing equation (3.3),

$$\begin{cases} [\frac{v^{\hat{\beta}}}{z}] TK[l_1(t)] = \frac{v^{\hat{\alpha}}}{z} l_0 \\ [\frac{v^{\hat{\beta}}}{z} + m_2] TK[l_2(t)] - m_1 TK[l_1(t)] = \frac{v^{\hat{\alpha}}}{z} 0 \end{cases} \tag{3.4}$$

$$H = \begin{bmatrix} \frac{v^{\hat{\beta}}}{z} + m_1 & 0 \\ -m_1 & \frac{v^{\hat{\beta}}}{z} + m_2 \end{bmatrix}$$

,

$$J = \begin{bmatrix} -\frac{v^{\hat{\alpha}}}{z} l_0 \\ 0 \end{bmatrix}.$$

Using cramer rule for solving equation (3.4),

$$\frac{\begin{bmatrix} \frac{v^{\hat{\alpha}}}{z} l_0 & 0 \\ 0 & \frac{v^{\hat{\beta}}}{z} + m_2 \end{bmatrix}}{\begin{bmatrix} \frac{v^{\hat{\beta}}}{z} + m_1 & 0 \\ -m_1 & \frac{v^{\hat{\beta}}}{z} + m_2 \end{bmatrix}} = \frac{\frac{v^{\hat{\alpha}}}{z} l_0}{\frac{v^{\hat{\beta}}}{z} + m_1} \tag{3.5}$$

$$\frac{\begin{bmatrix} \frac{v^{\hat{\beta}}}{z} + m_1 & \frac{v^{\hat{\alpha}}}{z} l_0 \\ -m_1 & 0 \end{bmatrix}}{\begin{bmatrix} \frac{v^{\hat{\beta}}}{z} + m_1 & 0 \\ -m_1 & \frac{v^{\hat{\beta}}}{z} + m_2 \end{bmatrix}} = \frac{m_1 \frac{v^{\hat{\alpha}}}{z} l_0}{(\frac{v^{\hat{\beta}}}{z} + m_1)(\frac{v^{\hat{\beta}}}{z} + m_2)}, \tag{3.6}$$

now apply Inverse Taki integral transforn in equations (3.5) and (3.6),

$$TK^{-1}\{TK[l_1(t)]\} = TK^{-1}\Big\{\frac{\frac{v^{\hat{\alpha}}}{z} l_0}{\frac{v^{\hat{\beta}}}{z} + m_1}\Big\}$$

$$l_1(t) = l_0 e^{-m_1 t} \tag{3.7}$$

$$TK^{-1}\{TK[l_2(t)]\} = TK^{-1}\Big\{\frac{m_1 \frac{v^{\hat{\alpha}}}{z} l_0}{(\frac{v^{\hat{\beta}}}{z} + m_1)(\frac{v^{\hat{\beta}}}{z} + m_2)}\Big\}$$

$$= TK^{-1}\Big[\frac{m_1 l_0}{m_2 - m_1}\Big\{\frac{\frac{v^{\hat{\alpha}}}{z}}{(\frac{v^{\hat{\beta}}}{z} + m_1)} - \frac{\frac{v^{\hat{\alpha}}}{z}}{(\frac{v^{\hat{\beta}}}{z} + m_2)}\Big\}\Big]$$

$$l_2(t) = \frac{m_1 l_0}{m_2 - m_1}\Big[e^{-m_1 t} - e^{-m_2 t}\Big] \quad (3.8)$$

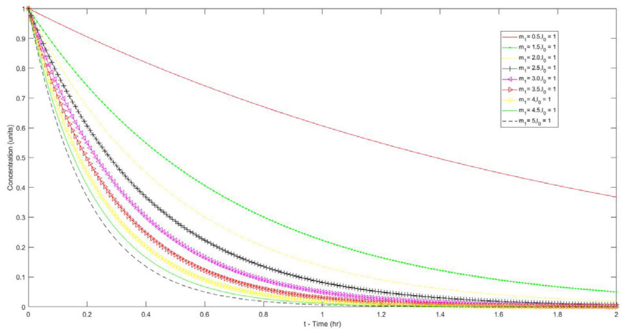
Accordingly we have found $l_1(t)$: concentration of drug in stomach or GI tract, $l_2(t)$: Absorption of drug in bloodstream sections.

Table 2.: Values of $l_1(t)$ at time t for different combinations of constants m_1 and l_0

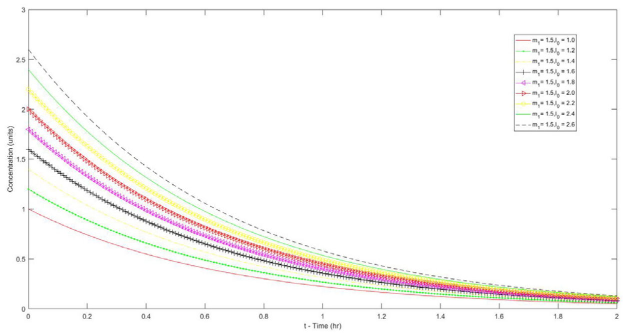
t(hr)	$l_1(t), l_0 = 1$								
	$m_1 = 0.5$	$m_1 = 1.5$	$m_1 = 2.0$	$m_1 = 2.5$	$m_1 = 3$	$m_1 = 3.5$	$m_1 = 4$	$m_1 = 4.5$	$m_1 = 5$
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.61	0.22	0.14	00.08	0.05	0.03	0.02	0.01	0.006
2.0	0.37	0.05	0.018	0.0068	0.002	0.0009	0.0003	0.0001	0.00005
3.0	0.22	0.01	0.002	0.0005	0.0001	0.00002	0	0	0
4.0	0.14	0.002	0.0003	0.00004	0	0	0	0	0
5.0	0.08	0.0005	0.00004	0	0	0	0	0	0

Table 3.: Values of $l_1(t)$at time t for different combinations of constants m_1 and l_0

t(hr)	$l_1(t), m_1 = 1.5$								
	$l_0 = 1.0$	$l_0 = 1.2$	$l_0 = 1.4$	$l_0 = 1.6$	$l_0 = 1.8$	$l_0 = 2.0$	$l_0 = 2.2$	$l_0 = 2.4$	$l_0 = 2.6$
0	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6
1.0	0.22	0.27	0.31	0.35	0.40	0.44	0.49	0.53	0.58
2.0	0.05	0.06	0.069	0.07	0.089	0.09	0.10	0.11	0.129
3.0	0.01	0.013	0.015	0.017	0.019	0.022	0.024	0.026	0.028
4.0	0.002	0.003	0.003	0.0039	0.004	0.004	0.005	0.005	0.006
5.0	0.0005	0.0007	0.0007	0.00088	0.0009	0.001	0.001	0.0013	0.0014



(a) For m_1 constant



(b) For a_1 constant

Figure 1.: Drug concentration pattern within the body as single compartment with different rate constants.

In table 2 if we fix l_0 and increase m_1 by 0.5 - 5 then there is a decrease in the value of $l_1(t)$ which is given as "a" in Fig.1. Similarly, In table 3, if we fix m_1 at 1.5 and increase l_0 from 1 - 2.6, the value of $l_1(t)$ decreases which is visible in "b" of fig 1. If we compare both the figs in Fig 1, the value of $l_1(t)$ decreases more when we fix l_0 and increase m_1.

Table 4.: Values of $l_2(t)$ at time t for different combinations of constants l_0, m_1 and m_2

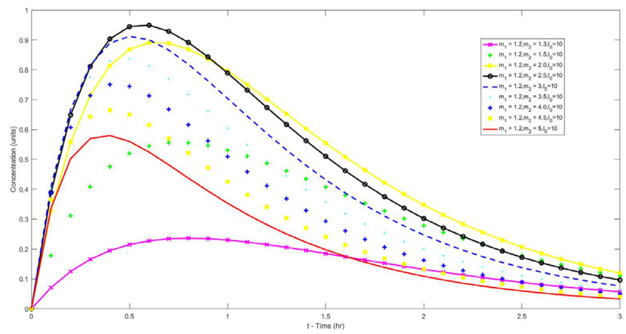
t (hr)	$l_2(t)$								
	$m_1 = 1.2, l_0 = 10$								
	$m_2 = 1.3$	$m_2 = 1.5$	$m_2 = 2.0$	$m_2 = 2.5$	$m_2 = 3.0$	$m_2 = 3.5$	$m_2 = 4.0$	$m_2 = 4.5$	$m_2 = 5$
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	3.43	3.12	2.4	2.02	1.67	1.41	1.21	1.05	0.92
2.0	1.97	1.63	1.08	0.77	0.58	0.46	0.38	0.32	0.28
3.0	0.84	0.64	0.37	0.24	0.18	0.14	0.11	0.09	0.08
4.0	0.32	0.23	0.11	0.075	0.05	0.04	0.03	0.029	0.02
5.0	0.11	0.077	0.03	0.022	0.01	0.01	0.01	0.009	0.007

Table 5.: Values of $l_2(t)$ at time t for different combinations of constants l_0, m_1, and m_2

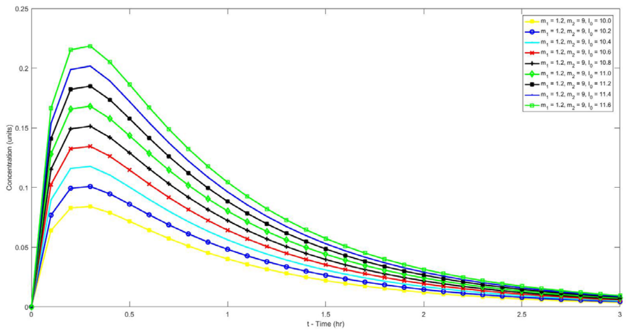
t (hr)	$l_2(t)$								
	$m_1 = 1.2, m_2 = 9$								
	$l_0 = 10$	$l_0 = 10.2$	$l_0 = 10.4$	$l_0 = 10.6$	$l_0 = 10.8$	$l_0 = 11$	$l_0 = 11.2$	$l_0 = 11.4$	$l_0 = 11.6$
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.46	0.47	0.48	0.49	0.50	0.51	0.52	0.52	0.53
2.0	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.16	0.16
3.0	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.04
4.0	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
5.0	0.003	0.003	0.004	0.004	0.004	0.004	0.004	0.004	0.004

Table 6.: Values of $l_2(t)$ at time t for different combinations of constants l_0, m_1 and m_2

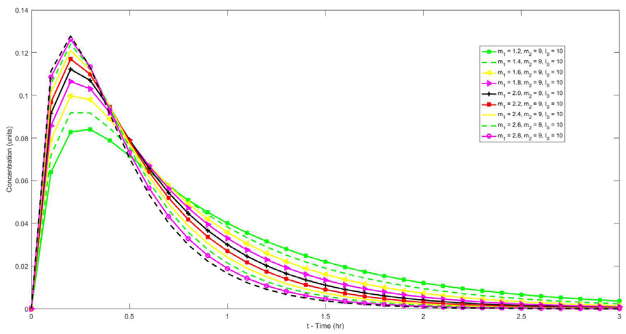
t (hr)	$l_2(t)$								
	$m_2 = 9, l_0 = 10$								
	$m_1 = 1.2$	$m_1 = 1.4$	$m_1 = 1.6$	$m_1 = 1.8$	$m_1 = 2.0$	$m_1 = 2.2$	$m_1 = 2.4$	$m_1 = 2.6$	$m_1 = 2.8$
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.46	0.45	0.43	0.41	0.38	0.35	0.33	0.30	0.27
2.0	0.14	0.11	0.08	0.068	0.05	0.03	0.029	0.02	0.02
3.0	0.04	0.027	0.017	0.011	0.007	0.004	0.003	0.002	0.001
4.0	0.01	0.006	0.003	0.002	0.0009	0.0005	0.0002	0.0001	0.00006
5.0	0.003	0.0016	0.0007	0.0003	0.0001	0.00005	0.00002	9.18259×10^{-6}	3.75529×10^{-6}



(a)



(b)



(c)

Figure 2.: Drug distribution pattern in oral administration.

In table 4, when we increase m_2 by 1.3 - 5 by fixing m_1 at 1.2 and l_0 at 10 then the value of $l_2(t)$ increases at $t = 1$ and then decreases which is shown in 'a' of Fig. 2.

In table 5, when we increase m_1 by 1.2 - 2.8 by fixing m_2 at 9 and l_0 at 10 then the value of $l_2(t)$ increases and then decreases at $t = 2$ which is shown in 'b' of Fig. 2.
In table 6, when we fix m_1 at 1.2 and m_2 at 9 and increase l_0 from 10 - 11.6, then the value of $l_2(t)$ is maximum at $t = 1$ and then it keeps decreasing as is visible in 'c' of Fig. 2. If we compare 'a', 'b' and 'c' of Fig. 2, then by fixing m_1 and m_2 and changing l_0, the result is good.

4. Conclusion

In the health sciences, we effectively use the Taki integral transform for medication absorption models in the tissue medium and blood compartments. The outcomes of Taki's integral transform are identical to those of Khanday et al. (2017) study [2] and P. Patil et al.'s study, which employed the Eigen value approach, Laplace transform, and General Integral Transform[1].Further research is needed to explore the full potential of the Taki transform in medical sciences and to compare its performance with existing integral transforms in various medical modeling scenarios.

References

[1] Patil, D. (2022), "General Integral Transform for the Solution of Models in Health Sciences." International Journal of Innovative Science and Research Technology **7**, 1177-1183.

[2] Khanday,et al. (2017), "Mathematical models for drug diffusion through the compartments of blood and tissue medium." Alexandria Journal of Medicine **53**, 245-249.

[3] Aitouni R.El , and Patil D. (2025), Taki transform and its applications, Interna tional Journal of Advances in Engineering and Management (IJAEM), **7**, 405-415.

[4] Al-Bugami et al. (2025), "El-Zaki Transform Method for Solving Applications of Some Integral Differential Equations." Contemporary Mathematics, 1693-1714.

[5] Thakur, A. K., and Kulmitra M. (2023)"New Results on Laplace Transform and its Applications." Journal website: www. internationaljournalsiwan. com, **57.**

[6] Jaiswal, Y., et al. (2025), "Applications of the Triple generalized Sumudu Laplace transform (TGSLT) to solve singular pseudo-hyperbolic equation using decompositions method." Mathematics in Engineering, Science and Aerospace (MESA) **16.**

[7] Thakur, A. K., R. Kumar, and Sahu. G. (2018), "Application of laplace transform on solution of fractional differential equation." Journal of Computer and Mathematical Sciences **9**, 478-484.

[8] Thakur, A. K., and S. Panda. (2015), "Some properties of triple Laplace transform." Journal of Mathematics and Computer Applications Research **2250**, 2408.

[9] Saadeh, et al.(2022), "A new approach using integral transform to solve cancer models." Fractal and Fractional **6**, 490.